Question

In the branching chain obtained when considering the disappearance of family lines suppose that the number of male offspring of any male individual has a binomial distribution with $n = 10$ and $p = \frac{1}{4}$, independently of the number of offspring of any other male. Find the 1-step transition probability p_{jk} for the total number of males in a generation.

Answer

Let Z_i be the number of male children produced by the i-th male in a generation.

Then $Z_i \sim B\left(10, \frac{1}{4}\right)$

If in generation m there are j males then the number of males in generation $m + 1$ is $X_{m+1} = Z_1 + ... + Z_j$

So X_{m+1} is a sum of j i.i.d. binomial r.v.'s and so $X_{m+1} \sim B\left(10j, \frac{1}{4}\right)$ so

$$p_{jk} = \begin{cases} \begin{pmatrix} 10j \\ k \end{pmatrix} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{10j-k} & \text{if } k \leq 10j \\ 0 & \text{otherwise} \end{cases}$$