Question

Determine whether or not the Markov chain with the following probability transition matrix, P, is ergodic i.e. possesses a limiting distribution independently of the initial distribution.

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

What conclusions do you draw?

Answer

The Markov chain is not irreducible.

The state space partitions into two closed irreducible sets of states: $\{1, 2\}$ and $\{3, 4\}$, so the limiting distribution will depend on the initial distribution. The Markov chains consisting of $(1, 2)$ and $(3, 4)$ are each irreducible and so all states are ergodic, since they are clearly aperiodic. So this example shows that finite aperiodic Markov chain's which are not irreducible need not be ergodic even if each of their states is ergodic. The stationary distribution not unique. The following is stationary for all values of p :

$$\begin{pmatrix} p \\ p \\ \frac{1}{2} - p \\ \frac{1}{2} - p \end{pmatrix}$$