Question
Let $\mathcal{M}$ be any σ-algebra of sets. Show that

i) $E \epsilon \mathcal{M},\ F \epsilon \mathcal{M} \Rightarrow E \cup F \epsilon \mathcal{M}$

ii) $E \epsilon \mathcal{M},\ F \epsilon \mathcal{M} \Rightarrow E \cap F \epsilon \mathcal{M}$

iii) $E \epsilon \mathcal{M},\ F \epsilon \mathcal{M} \Rightarrow E - F \epsilon \mathcal{M}$

iv) $\phi \epsilon \mathcal{M}$

v) $E_1, E_2, \cdots E_n \cdots \epsilon \mathcal{M} \Rightarrow \bigcap_{n=1}^{\infty} E_n \epsilon \mathcal{M}$

Answer
$\mathcal{M}$ is a σ-algebra of sets. i.e.

a) $E \epsilon \mathcal{M} \Rightarrow E^C \epsilon \mathcal{M}$

b) $E_1, E_2 \cdots \epsilon \mathcal{M} \Rightarrow \bigcup_{i=1}^{\infty} E_i \epsilon \mathcal{M}$

i) Suppose $E, F \epsilon \mathcal{M}$. Let $E_1 = E,\ E_2 = E_3 = \cdots = F$.
Then $\bigcup_{i=1}^{\infty} E_i = E \cup F \epsilon \mathcal{M}$.

ii) $E \cap F = (E^C \cup F^C)^C \epsilon \mathcal{M}$

iii) $E, F \epsilon \mathcal{M} \Rightarrow E \cap F^C = E - F \epsilon \mathcal{M}$

iv) $E \epsilon \mathcal{M} \Rightarrow E^C \epsilon \mathcal{M} \Rightarrow E \cap E^C = \phi \epsilon \mathcal{M}$

v) $E_1, E_2 \cdots E_n \cdots \epsilon \mathcal{M} \Rightarrow E_1^C, E_2^C \cdots E_n^C \cdots \epsilon \mathcal{M}$
$$\Rightarrow \left(\bigcup_{i=1}^{\infty} (E_i)^C \right)^C \epsilon \mathcal{M} \Rightarrow \bigcap_{i=1}^{\infty} E_i \epsilon \mathcal{M}$$