QUESTION

(i) Find the vector equation of the line L that passes through the two points $A(1,-2,1)$ and $B(2,3,1)$.

(ii) A plane P passes through the three points $C(2,1,-3)$, $D(4,-1,2)$ and $E(3,0,1)$. Obtain two independent vectors that are parallel to P and hence, or otherwise, show that P has the vector equation

$$\mathbf{r}.(1,1,0) = 3$$

(iii) Find the coordinates of the point of intersection of the line L and the plane P.

(iv) Derive the vector equation of the plane that contains the line L and is perpendicular to the plane P.

ANSWER

(i) $\vec{AB} = (2-1, 3-(-2), 1-1) = (1,5,0)$. So the equation of L is $\mathbf{r} = (1,-2,1) + s(1,5,0) = (1+s, -2+5s, 1)$

(ii) $\vec{CD} = (4-2, -1-1, 2-(-3)) = (2,-2,5)$
$\vec{CE} = (3-2, 0-1, 1-(-3)) = (1,-1,4)$
$\mathbf{n} = \vec{CD} \times \vec{CE} = (-8-(-5), 5-8, -2-(-2)) = (-3,-3,0)$
The equation of the plane is $\mathbf{r.n} = c$ i.e. $\mathbf{r}.(-3,-3,0) = c$

$C(2,1,-3)$ lies on the plane,

therefore $c = (2,1,-3).(-3,-3,0) = -6-3+0 = -9$.

So the equation of the plane is $\mathbf{r}.(-3,-3,0) = -9$ or $\mathbf{r}.(1,1,0) = 3$.

(iii) The line L meets the plane P when $(1+s, -2+5s, 1).(1,1,0) = 3$. Therefore $1+s-2+5s+0 = 6s-1 = 3$, with solution $s = \frac{2}{3}$, so the point of intersection $= \left(\frac{5}{3}, \frac{4}{3}, 1\right)$

(iv) We need to find a second plane parallel to L and $\mathbf{n}$.
The normal vector is in the direction $(1,5,0) \times (1,1,0) = (0,0,-4)$
The equation of the plane is $\mathbf{r}.(0,0,-4) = k$.

A is on the plane, so $k = (1,-2,1).(0,0,-4) = -4$.

Hence the equation of the plane is $\mathbf{r}.(0,0,-4) = -4$ or $\mathbf{r}.(0,0,1) = 1$