NOTE REFERENCE TO QUESTION 1

Question

Verify the following integral.

$\int_0^\infty dx \frac{\cos mx}{x^2+1} = \frac{\pi}{2}\exp(-m),\ m > 0$

Answer

Consider $J = \oint_C \frac{dz}{e^{imz}}(z^2+1)$ where C is the D-contour of Q1.

The integrand has simple poles at $z = \pm i$, but only $z = i$ lies in C.

Residue at $z = i$ is $\lim_{z\to i}\left\{\frac{(z-i)e^{imz}}{(z-i)(z+i)}\right\} = \frac{e^{-m}}{2i}$

Then

$$J = 2\pi i \frac{e^-m}{2i} = \pi e^{-m}$$

Now $J = \int_{-RPICTURE}^{+R} \frac{dxe^{imx}}{x^2+1} + \int \frac{dze^{imz}}{z^2+1} = \pi e^{-m}$

or

$J = \int_{-R}^{+R} \frac{\cos mx}{x^2+1}\, dx + i\int_{-R}^{+R} \frac{\sin mx}{x^2+1}\, dx + \int_{PICTURE} \frac{dze^{imz}}{z^2+1} = \pi e^{-m}$

Now sin is an <u>odd</u> function so $\int_{-R}^{+R} \sin mx \cdots = 0$

Also as $R \to \infty \int_{PICTURE} \to 0$ so

$$\begin{aligned} \lim_{R\to\infty} J &= \lim_{R\to\infty} \int_{-R}^{+R} \frac{\cos mx}{x^2+1} dx = \pi e^{-m} \\ &\Rightarrow 2\int_0^\infty \frac{\cos mx}{x^2+1} dx = \frac{\pi e^{-m}}{2} \end{aligned}$$