NOTE REFERENCE TO QUESTION 1

Question

Verify the following integral.

$$\int_{-\infty}^{+\infty} \frac{x^2\,dx}{(x^2+1)^2(x^2+2x+2)} = \frac{7\pi}{50}$$

Answer

Consider $\oint_C \frac{dzz^2}{(z^2+1)^2(z^2+2Z+2)}$ around contour of Q1.

Integrand has poles at $\begin{array}{r} z^2+1=0 \quad\Rightarrow\quad z=\pm i \\ z^2+2z+2=0 \quad\Rightarrow\quad z=-1\pm i \end{array}$

$z=+i,\ z=-1+i$ are inside

$J = 2\pi i\times$ [(residue at $+i$)+(residue at $-1+i$)]

$$\begin{aligned} &\text{Residue at } \underline{\text{double}} \text{ pole} + \text{i} \\ = &\lim_{z\to+i}\left[\frac{1}{1!}\frac{d}{dz}\left\{\frac{(z-i)^2z^2}{(z-i)^2(z+i)^2(z^2+2z+2)}\right\}\right] \\ = &\lim_{z\to+i}\left[\frac{d}{dz}\left\{\frac{z^2}{(z+i)^2(z^2+2z+1)}\right\}\right] \\ = &\frac{9i-12}{100} \end{aligned}$$

$$\begin{aligned} &\text{Residue at } \underline{\text{simple}} \text{ pole} - 1 + \text{i} \\ = &\lim_{z\to-1+i}\left[\frac{(z+1-i)z^2}{(z+1)^2(z+1-i)(z-1+i)}\right] \\ = &\frac{3-4i}{25} \end{aligned}$$

Thus $J = 2\pi i\left(\frac{9i-12}{100}+\frac{3-4i}{25}\right) = \frac{7\pi}{50}$

Now look at $R\to\infty$. Contribution around semicircle vanishes to give

$$\int_{-\infty}^{+\infty} \frac{dxx^2}{(x^2+1)^2(x^2+2x+2)} = \frac{7\pi}{50}$$

as required.