Applications of Partial Differentiation
Extremes within restricted domains

Question
Find the maximum and minimum values of

$$f(x, y) = x - x^2 + y^2$$

On the rectangle $0 \le x \le 2$, $0 \le y \le 1$.
Answer
For critical points

$$\begin{aligned} 0 &= f_1(x, y) = 1 - 2x \\ 0 &= f_2(x, y) = 2y \end{aligned}$$

So the only CP is $(\frac{1}{2}, 0)$. This lies on the boundary of the rectangle. This boundary has four segments:
On $x = 0$

$$\begin{aligned} f(x, y) &= f(0, y) = y^2 \\ \text{for } 0 \le y \le 1 & \end{aligned}$$

This has min=0 and max=-1.
On $y = 0$

$$\begin{aligned} f(x, y) &= f(x, 0) = x - x^2 = g(x) \\ \text{for } 0 \le x \le 2 & \end{aligned}$$

Since $g'(x) = 1 - 2x = 0$ at $x = \frac{1}{2}$,

$$\begin{aligned} g(1/2) &= 1/4 \\ g(0) &= 0 \\ g(2) &= -2 \end{aligned}$$

This has min=-2 and max=1/4.
On $x = 2$

$$\begin{aligned} f(x, y) &= f(2, y) = -2 + y^2 \\ \text{for } 0 \le y \le 1 & \end{aligned}$$

This has min=-2 and max=-1.

On $y = 1$

$$\begin{aligned} f(x,y) &= f(x,1) = x - x^2 + 1 = g(x) + 1 \\ \text{for } 0 \leq x \leq 2 \end{aligned}$$

This has min=-1 and max=5/4.
So on the rectangle, f has
minimum value=-2, maximum value=5/4.