Question
Is it true that $\{l+o(1)\}\cosh x-\{1+o(1)\}\sinh x=\{1+o(1)\}e^{-x}$, as $x\to\infty$?

Answer
No!!

$$\begin{aligned}
& [1+o(1)]_{(1)}\cosh x-[1+o(1)]_{(2)}\sinh x \\
= & \frac{1}{2}\left\{[1+o(1)]_{(1)}e^{x}+[1+o(1)]_{(1)}e^{-x}\right\} \\
& -\frac{1}{2}\left\{[1+o(1)]_{(2)}e^{x}-[1+o(1)]_{(2)}e^{-x}\right\} \\
= & \frac{1}{2}[o(1)_{(1)}-o(1)_{(2)}]e^{x} \\
& +\frac{1}{2}\left\{[1+o(1)]_{(1)}+[1+o(1)]_{(2)}\right\}e^{-x} \\
= & o(e^{x}),
\end{aligned}$$

since $o(1)_{(1)}\neq o(1)_{(2)}$ necessarily,
e.g., $\frac{1}{x}\neq\frac{1}{x^2}$ as $x\to+\infty$ so it *doesn't* vanish.

($\star$) To show $o(1)-o(1)=o(1)$, let $f(x)=o(1),\ g(x)=o(1)$.
Then $f(x)\le K_f,\ g(x)\le K_g \quad K_f,\ K_g>0,\ x\to+\infty$
Then

$$\begin{aligned}
|f(x)-g(x)| &\le |f(x)|+|g(x)| \text{ by triangle inequality} \\
&\le K_f+K_g=K \text{ say as } \ x\to+\infty
\end{aligned}$$

Therefore $|f(x)-g(x)|\le K\Rightarrow f-g=o(1)\Rightarrow o(1)-o(1)-o(1)$
Similarly $o(1)+o(1)=o(1)$.

($\star\star$) To show $o(1)e^{x}+[1+o(1)]e^{-x}=o(e^{x})$
Clearly $e^{-x}=o(e^{x})$ as $x\to+\infty$
Therefore we must show that $o(1)e^{x}=o(e^{x}) \quad x\to+\infty$
Let $h(x)=o(1),\quad h>0,\ x\to\infty\Rightarrow\lim_{x\to\infty}h(x)=0$
Therefore $\lim_{x\to\infty}\frac{h(x)e^{x}}{e^{x}}=\lim_{x\to\infty}h(x)=0\Rightarrow o(1)e^{x}=o(e^{x})$