

**Question**

Determine the order of the following expressions as  $z \rightarrow +\infty$ , giving suitable implied constants.

(a)  $4\pi^2 z$

(b)  $1000z^{\frac{1}{2}}$

(c)  $\sqrt{z\left(1 - \frac{1}{z}\right)}$

**Answer**

(a)  $\lim_{z \rightarrow \infty} \frac{|4\pi^2 z|}{|z|} = 4\pi^2 \Rightarrow 4\pi^2 z = O(z), z \rightarrow +\infty, \underline{(k \geq 4\pi^2, \text{ say})}$

(b)  $\lim_{z \rightarrow +\infty} \frac{|1000z^{\frac{1}{2}}|}{|z^{\frac{1}{2}}|} = 1000$   
 $\Rightarrow 1000z^{\frac{1}{2}} = O(z^{\frac{1}{2}}), z \rightarrow +\infty, \underline{(k \geq 1000, \text{ say})}$

(c)  $\lim_{z \rightarrow +\infty} \frac{\left|z\sqrt{\left(1 - \frac{1}{z}\right)}\right|}{|\sqrt{z}|} = 1$   
 $\Rightarrow \sqrt{z\left(1 - \frac{1}{z}\right)} = O(z^{\frac{1}{2}}), z \rightarrow +\infty, \underline{(k \geq 1, \text{ say})}$